

3E1456

Roll No. _____

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3E1456**B.Tech. IIIrd Semester (Main/Back) Examination, Feb. - 2011****3CE6 Engineering Mathematics****Time : 3 Hours****Maximum Marks : 80****Min. Passing Marks : 24****Instructions to Candidates:**

Attempt **five** questions in all selecting **one** question from **each** unit. Schematic diagrams must be shown wherever necessary. Any data you feel missing may suitably be assumed and stated clearly. Units of quantities used/calculated must be stated clearly.

Unit - I

1. a) Find the Fourier series to represent $f(x) = x \cos x, -\pi < x < \pi$
 b) Obtain the first three cosine terms and the constant term in the Fourier series for y , where

$x:$	0	1	2	3	4	5
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$y:$	4	8	15	7	6	2
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OR

- a) Find the half range cosine series for the following function
 $f(x) = (x-1)^2, 0 < x < 1,$

hence show that

$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots$$

- b) The following table gives the variation of a periodic current over a period

t(secs)	:	0	$\frac{T}{6}$	$\frac{T}{3}$	$\frac{T}{2}$	$\frac{2T}{3}$	$\frac{5T}{6}$	T
A (amps)	:	1.98	1.30	1.05	1.30	-0.88	-0.25	1.98

Show by harmonic analysis that there is a direct current part of 0.75 amp. in the variable current and obtain the amplitude of the first harmonic.

Unit - II

2. a) If $L\{f(t)\} = \bar{f}(s)$, then prove that

$$L\{t f(t)\} = -\frac{d}{ds} \bar{f}(s)$$

Hence obtain $L\{t e^{at} \cos bt\}$.

- b) Solve by the Laplace transform theory the equation

$$\frac{\partial^2 y}{\partial t^2} = 9 \frac{\partial^2 y}{\partial x^2}, \quad y(0, t) = 0, \quad y(2, t) = 0,$$

$$y(x, 0) = 20 \sin 2\pi x - 10 \sin 5\pi x \text{ and } y_t(x, 0) = 0.$$

OR

- a) Apply the convolution theorem to obtain $L^{-1} \left\{ \frac{s}{(s^2 + a^2)^2} \right\}$.

- b) Use Laplace transform theory to solve the differential equation:

$$(D^2 + 1)x = t \cos 2t, \quad x(0) = 0, \quad x'(0) = 0.$$

Unit - III

3. a) Find the Fourier Sine and cosine transform of $f(x) = \begin{cases} 1, & \text{for } 0 < x < a \\ 0, & \text{for } x > a \end{cases}$.

- b) Solve:

$$\frac{\partial u}{\partial x} = \frac{\partial^2 u}{\partial t^2},$$

given that $u_x(0, t) = 0$ and $u(x, 0) = \begin{cases} x, & 0 \leq x \leq 1 \\ 0, & x > 1 \end{cases}$, $u(x, t)$ is bounded and $x > 0$, $t > 0$.

OR

- a) Find the Fourier transform of

$$f(x) = \begin{cases} x^2, & \text{when } |x| \leq a \\ 0, & \text{when } |x| > a \end{cases}$$

Hence evaluate

$$\int_0^{\infty} \cos \frac{as}{2} \left[(a^2 s^2 - 2) \sin as + 2as \cos as \right] / s^3 ds.$$

- b) Using Fourier sine transform, solve the differential equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \quad x > 0, \quad t > 0 \text{ subject to the conditions :}$$

$$u(0, t) = 0, \quad u(x, 0) = \begin{cases} 1, & \text{when } 0 < x < 1 \\ 0, & \text{when } x \geq 1 \end{cases}$$

It may be assumed that $u(x, t)$ is bounded; also u and $\frac{\partial u}{\partial x}$ approach zero as $x \rightarrow \infty$

Unit-IV

4. a) Prove that

$$i) \quad u_0 + \frac{x u_1}{1} + \frac{x^2}{2} u_2 + \frac{x^3}{3} u_3 + \dots = e^x \left[u_0 + x \Delta u_0 + \frac{x^2}{2} \Delta^2 u_0 + \dots \right]$$

$$ii) \quad u_1 x + u_2 x^2 + u_3 x^3 + \dots = \frac{x}{1-x} u_1 + \left(\frac{x}{1-x} \right)^2 \Delta u_1 + \left[\frac{x}{1-x} \right]^3 \Delta^2 u_1 + \dots$$

- b) A slider in a machine moves along a fixed straight rod, Its distance x (cm) along the rod is given below for various values of time t (sec s).

$t:$	0	0.1	0.2	0.3	0.4	0.5	0.6
$x:$	30.28	31.43	32.98	33.54	33.97	33.48	32.13

Evaluate i) $\frac{dx}{dt}$ for $t=0.1$

and ii) $\frac{dx}{dt}$ for $t=0.3$

OR

- a) i) Evaluate $\int_{-1.6}^{-1} e^x dx$ by Simpson's one third rule with six intervals.

ii) Prove that $\nabla \Delta = \partial^2 = \Delta - \nabla$

- b) Use Stirling's formula to compute $U_{12.2}$ from the following data:

$x:$	10	11	12	13	14
$10^5 u_x:$	23967	28060	31788	35209	38368

Unit - V

5. a) Employ Euler's method to solve :

$$\frac{dy}{dx} = \frac{y^2 - x}{y^2 + x}, \text{ given } y = 1, x = 0.$$

Find y for $x = 0.1, 0.2$ and 0.3 .

- b) Use Milne's predictor - corrector method to find the solution of the differential equation

$$\frac{dy}{dx} = x - y^2$$

for next value of x , given that

$$y(0) = 0.0000, \quad y(0.2) = 0.0200,$$

$$y(0.4) = 0.0795, \quad y(0.6) = 0.1762,$$

OR

- a) Use Picard's method to solve

$$\frac{dy}{dx} = 1 + xy, \text{ with } x_0 = 2, y_0 = 0 \text{ up to third Order of approximation.}$$

- b) Using Runge-Kutta fourth order method, find the approximate value of

$$y \text{ for } x = 0.2 \text{ if } \frac{dy}{dx} = x + y^2 \text{ given that } y = 1 \text{ when } x = 0, \text{ step size } h = .1.$$